

# Generation of toroidal magnetic fields in accretion disks

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## ABSTRACT

Magnetic fields play an important role in the dynamics of accretion disks, however, the origin of the fields is often obscured. Here we show that magnetic fields can be generated in an initially non-magnetized accretion disks through the Biermann battery mechanism, where the radial temperature profile and the vertical density profile of these systems provide the necessary conditions for this process to operate naturally. We consider the generation of fields in a protoplanetary disks and disks around Black Holes (BHs). For protoplanetary accretion disks we find that the generated magnetic fields can be as strong as few Gauss over the lifetime of the disk (5 Myr), for a solar mass star, and weakly dependent on the accretion rate. The generated seeds have toroidal structure with opposite sign in the upper and lower half of the disk. The same pattern exists in a thin accretion disk around a rotating BH, where the field generation rate increases for larger BH's spin parameters in a co-rotating disk and spin configuration. At a fixed  $r/r_{\text{isco}}$ , where  $r$  is the radial distance from the BH, the battery scales as  $M^{-2}$  for both thin accretion disks and Advection Dominated Accretion Flows (ADAF). For ADAFs after one characteristic accretion time the battery can generate a field in the galactic center at the order of  $5 \times 10^{-6}$  G. In addition, we test this mechanism in GRMHD simulations and find it to be in a good agreement with our analytical estimates. The fact that the battery works in the limit of zero accretion rate, it makes this mechanism a viable candidate to provide the seed fields from an initially non-magnetized accretion disk such that later on Magneto Rotational Instability (MRI) takes over.

**Key words:** accretion, accretion discs — dynamo — instabilities — magnetic fields — MHD

## 1 INTRODUCTION

Accretion disks are associated with a wide range of astrophysical phenomena from disks around protostars to Black Holes (BHs). These disks are assumed to be magnetized and the accretion to happen through the Magneto Rotational Instability (MRI) process (Balbus & Hawley 1991). However, the origin of the magnetic field in the disk is rarely discussed. There are a few approaches that are adopted in the literature to address the source of magnetic field in accretion disks. For the stellar mass BHs, the magnetic field is typically associated with the magnetic field of the progenitor star (e.g., Bisnovatyi-Kogan & Ruzmaikin 1974, 1976), while for a super-massive BHs the field is believed to be accreted from the surrounding gas (Narayan et al. 2003; Eatough et al. 2013). The magnetic fields around the first generation of protostars can be the result of large scale cascades, where weak magnetic fields are amplified via

turbulent dynamo during the gravitational collapse for the first generation of protostars (e.g., Schleicher et al. 2010; Schober et al. 2012). Although these mechanisms can play a role in providing the seed magnetic fields, here we propose that magnetic fields in accretion disks can be naturally formed through the Biermann battery process (Biermann 1950).

Biermann (1950) showed that if a plasma has a rotational motion, current must exist, which leads to the generation of magnetic fields. This process has a wide range of applications from the generation of magnetic fields in stars (e.g., Biermann 1950; Doi & Susa 2011) to galactic scale magnetic fields (e.g., Mestel & Roxburgh 1962; Widrow 2002; Subramanian et al. 1994; Subramanian 2010; Naoz & Narayan 2013). For example, Naoz & Narayan (2013) showed that magnetic fields are naturally generated in the early universe, at intergalactic scales, which is consistent with the observed fields' strength. The Biermann battery was also shown to be able to generate magnetic fields inside supernova bubbles (e.g., Hanayama et al. 2005).

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For the generation of seed magnetic fields in this mechanism, a nonaligned gradient of electrons density and gradient of electrons pressure is needed. This situation naturally arises in the structure of accretion disks where the pressure gradient is mostly in the vertical direction and temperature has a radial gradient in the disk. These seed magnetic fields can grow in linear timescales and in a differentially rotating disk can lead to a strong magnetohydrodynamic (MHD) instability through MRI that makes the field to grow exponentially with time to reach its equipartition value in the disk (e.g., Bret 2009). Here we focus on generating seed magnetic field in an initially non-magnetized accretion disk. We do not specify the nature that causes the accretion and we assume that accretion takes place while the disk is *not* initially magnetized (i.e., MRI does not operate). Turbulent viscosity could be thought of as a plausible mechanism for our purposes (e.g., Shakura & Sunyaev 1973). As we will show below, generation of the seed magnetic fields through Biermann battery depends only very weakly on the accretion rate. Therefore, apriori we do not require MRI to be taking place to provide accretion for the battery to work.

Shiromoto et al. (2014) studied the generation of magnetic fields in a first generation proto-circumstellar disk based on 2D radiation hydrodynamics simulations. Unlike their work, here we study a range of applications from generation of seed magnetic fields around a protostar, to supermassive rotating black hole at the center of the galaxy.

The paper is organized as follow: In §2 we introduce the Biermann battery equation. In §3 we apply the Biermann battery equation in protoplanetary disk setting. In §4 the battery is applied in thin accretion disk around Kerr black holes. In §5 we compare the rates predicted for ADAFs with ones obtained for state-of-the-art general relativistic (GR) MHD simulations of radiatively inefficient accretion flows and In §6 we summarize and give conclusions.

## 2 BIERMANN BATTERY

The generation of magnetic fields in Biermann battery is due to non-vanishing cross product of electron number density gradient,  $\nabla n_e$  and electron pressure gradient,  $\nabla P_e$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - c \frac{\nabla n_e \times \nabla P_e}{en_e^2}. \quad (1)$$

Assuming ideal gas equation of state,  $P_e = n_e k_B T_e$  and assuming that the temperature of the electrons is equal to that of the gas,  $T_e = T_{\text{disk}}$  we can re-write  $\nabla n_e \times \nabla P_e$  as

$$\nabla n_e \times \nabla P_e = \nabla n_e \times (n_e \nabla T_e + T_e \nabla n_e) k_B, \quad (2)$$

where  $k_B$  is Boltzman constant. Given  $n_e = \chi_e \rho$ , where  $\chi_e$  is the ionization fraction, we find

$$\nabla n_e \times \nabla P_e = n_e k_B \nabla n_e \times \nabla T_e = n_e k_B \chi_e \nabla \rho \times \nabla T_e, \quad (3)$$

Where without the induction term we have

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{ck_B}{e} \frac{\nabla \rho \times \nabla T_e}{\rho} \quad (4)$$

Note that the ionization fraction,  $\chi_e$ , cancels out and thus the equation depends only on the density and temperature of the system.

## 3 PROTOPLANETARY DISK

Planets are formed from the protoplanetary disks made of gas and dust surrounding young stars. These disks are not fully ionized,

as the temperatures of the disks are estimated from few thousands Kelvin in the inner regions of the disk down to tens of Kelvin in the outer parts. However, as mentioned above, the Biermann battery is independent on the ionization fraction. We adopt a simple density and temperature profile for a geometrically thin, Keplerian protoplanetary disk, (for review of the derivation see Armitage 2013). The axisymmetric disk is described by its vertical and radial structure. Assuming a hydrostatic equilibrium the vertical structure is determined by the pressure gradient equation in the  $\hat{z}$  direction

$$\frac{dP}{dz} = -\rho \Omega_K^2 z, \quad (5)$$

where  $\Omega_K$  is the Keplerian frequency defined as:

$$\Omega_K = \sqrt{\frac{GM}{r^3}} \quad (6)$$

Assuming an ideal gas equation of state, the density profile in the vertical direction is

$$\rho = \frac{\Sigma}{\sqrt{2\pi}H} e^{-\frac{z^2}{2H^2}}, \quad (7)$$

With the scale height defined as  $H = c_s/\Omega_K$  and the speed of sound has only a radial dependency (e.g., Armitage 2013)

$$c_s = \sqrt{\frac{k_B T(r)}{\mu m_p}}. \quad (8)$$

The surface density,  $\Sigma$ , is given by

$$\Sigma(r) = \frac{\dot{M}}{3\pi\nu} \left(1 - \sqrt{\frac{R_*}{r}}\right), \quad (9)$$

where  $\nu = H c_s \alpha$  (Shakura & Sunyaev 1973).  $\alpha$  is a dimensionless parameter and our results does not depend on its specific value.

The temperature radial profile is determined by the accretion rate and the balance of heating and cooling in the disk which leads to radial effective temperature profile of

$$T_{\text{eff}}^4(r) = \frac{3GM\dot{M}}{8\pi\sigma r^3} \left(1 - \sqrt{\frac{R_*}{r}}\right). \quad (10)$$

where  $\sigma$  is the Boltzmann constant. We compute the battery at  $z = H$  for all the calculations. To keep the analysis simple, we set  $T_e \sim T_{\text{eff}}$  in eq (4) and  $T \sim T_{\text{eff}}$  in eq (8).<sup>1</sup> Since  $\nabla \rho$  and  $\nabla T_e$  both have  $\hat{r}$  and  $\hat{z}$  components, the field is generated in the toroidal direction, i.e.,

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{ck_B}{e\rho} \left( \frac{\partial T_e}{\partial r} \frac{\partial \rho}{\partial z} - \frac{\partial \rho}{\partial r} \frac{\partial T_e}{\partial z} \right) \hat{\phi} \quad (11)$$

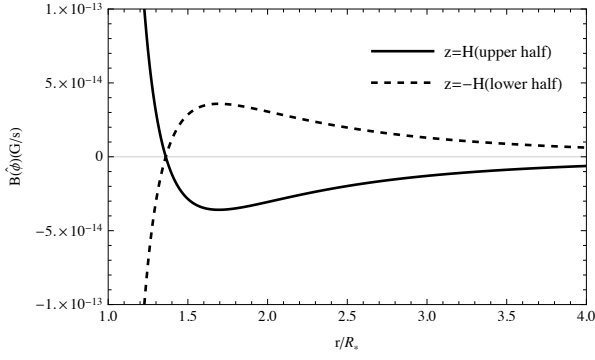
The Biermann battery will work so long as the density and temperature gradients are not parallel to each other. This condition will almost always be satisfied in a disk. To avoid getting into details, we estimate the order of magnitude of the Biermann battery by crossing the vertical gradient of density with the radial gradient of temperature. Therefore Equation (11) can be written as:

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{ck_B}{e\rho} \left( \frac{\partial T_e}{\partial r} \frac{\partial \rho}{\partial z} \right) \hat{\phi}, \quad (12)$$

which, can be written as:

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{cGMm_p\mu}{8e} \frac{7R_* + r(\sqrt{R_*}/r - 6)}{r^4(r - R_*)} z \hat{\phi} \quad (13)$$

<sup>1</sup> The disk mid-plane temperature is often just a factor of a few larger than  $T_{\text{eff}}$ . We feel therefore that this approximation is reasonable.



**Figure 1.** Magnitude of the rate of magnetic field generation as a function of  $r/R_*$  for a  $1 M_\odot$  star at  $z = H$  on the disk. Generated fields change their sign at  $r_{\text{turn}} = 49/36 R_*$ . The seeds have an opposite sign on the upper and lower half of the disk. The rate of seed generation through Biermann battery for a protoplanetary disk is proportional to the accreting mass  $\dot{M}$  and only weakly dependent on the accretion rate ( $\propto \dot{M}^{1/8}$ ). We adopt  $\mu = 2.3$  in this calculation.

Note that the magnetic field changes its azimuthal direction going from the upper to the lower half of the disk. There is no dependence on accretion rate in this equation, however, as we compute the battery at  $z = H$ , it would mean a weak dependence on accretion rate ( $\propto \dot{M}^{1/8}$ ).

The magnetic field generated over an accretion timescale ( $r/u_r$ ) is at the order of  $50 \mu\text{G}$  for a  $1 M_\odot$  star with  $R_* = 1 R_\odot$ . The battery can operate over the lifetime of the disk and can reach about few to tens of Gauss in large parts of the disk. In Figure 1 we show the seeds generation rate in a protoplanetary disk. The vertical distance adopted in our calculations scales as the scale height, i.e.,  $z = H$ . The generated seeds change their sign from upper half to lower half of the disk and also at the characteristic radius  $r_{\text{turn}} = 49/36 R_*$ .

We note that the change of azimuthal sign of  $\partial B/\partial t$  at  $r_{\text{turn}}$  arises because  $T_e$  has a maximum at this radius. This is predicted by the thin disk model, but whether this exact shape is followed by real disks is disputed.

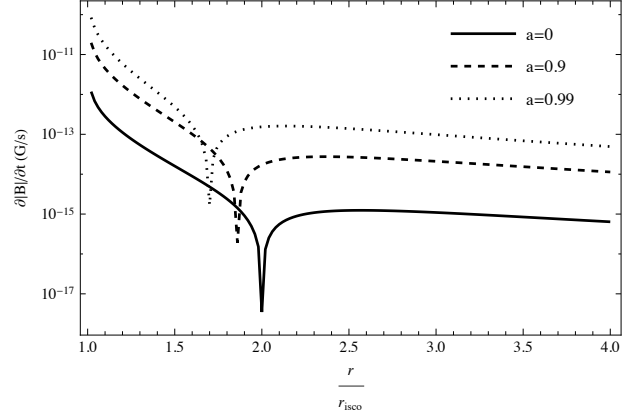
The equipartition field can be estimated by equating the magnetic pressure  $\sim B^2/8\pi$  and the pressure of the ionized gas. For protoplanetary disks it is in the order of  $10^3 \text{ G}$  at the vicinity of  $R_*$ , which is orders of magnitude larger than the Biermann seed field. Furthermore, we also estimate when MRI starts to dominated the accretion process. Taking the most unstable wavelength,  $\sim 2\pi v_A/\Omega$ , where  $v_A = B/\sqrt{4\pi\rho^2}$  is the Alfvén wave velocity, and equating it to  $H = c_s/\Omega$ , for  $z \sim H$ , we find magnetic field peaking at  $r/R_\odot \sim 1.5$ , with strength of  $\sim 200 \text{ G}$ , which is order of magnitude larger than the Biermann field at this radius after 5 Myr, and stays in similar order of magnitude difference at all radii.

#### 4 SEEDS IN DISK AROUND KERR BLACK HOLES

Following Shakura & Sunyaev (1973) and Novikov & Thorne (1973), the vertical density and radial temperature profiles of these disks is described by similar equations as presented above, with the following relativistic corrections:

$$\frac{dP}{dz} = -\rho \frac{GM}{r^3} \frac{C}{\mathcal{B}}, \quad (14)$$

<sup>2</sup> For simplicity we assume a completely ionized gas.



**Figure 2.** Shows the magnitude of magnetic field generation rate as a function of  $r/r_{\text{isco}}$  for a  $4 \times 10^6 M_\odot$  black hole with accretion rate of  $\dot{M} = 10^{-10} M_\odot/\text{year}$  for different spin parameters. The battery is about 2 orders of magnitude stronger in  $a=0.99$  case compared with a Schwarzschild black hole at all radii. The radius at which the generated seed fields change sign in azimuthal direction ( $r_{\text{turn}}$ ) becomes smaller as the spin parameter is increased.  $r_{\text{turn}} = 2 \times r_{\text{isco}}$  for a Schwarzschild black hole. We set  $z = H$  in this calculation. The field changes its sign across the equatorial plane in  $\phi$  direction.

which assuming a hydrostatic equilibrium leads to the following density profile

$$\rho = \frac{\Sigma}{\sqrt{2\pi H}} e^{\frac{-2}{2H^2} \frac{C}{\mathcal{B}}} \sqrt{\frac{C}{\mathcal{B}}}, \quad (15)$$

where

$$\Sigma = \frac{\dot{M}}{3\pi\nu} \left(1 - \sqrt{\frac{r_{\text{isco}}}{r}}\right), \quad (16)$$

and  $H, \nu, \Sigma$  are a function of  $r$  and  $a$  where  $a$  is specific angular momentum of the black hole.

The temperature profile from the balance of heating and cooling in the disk is

$$T_c^4(r) = \frac{3GM\dot{M}}{8\pi\sigma r^3} \left(1 - \sqrt{\frac{r_{\text{isco}}}{r}}\right) \frac{\mathcal{D}}{\mathcal{B}} \quad (17)$$

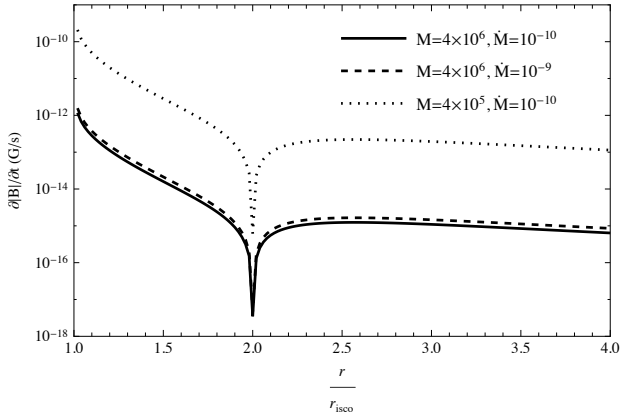
$\mathcal{A}, \mathcal{B}, \mathcal{C}$  and  $\mathcal{D}$  are relativistic correction (Novikov & Thorne 1973; Page & Thorne 1974; Doerr et al. 1996) defined as:

$$\begin{aligned} \mathcal{A} &= 1 - \frac{2GM}{c^2 r} + \frac{a^2}{c^2 r^2} \\ \mathcal{B} &= 1 - \frac{3GM}{c^2 r} + \frac{2a\sqrt{GM}}{c^2 r^{3/2}} \\ \mathcal{C} &= 1 - \frac{4a\sqrt{GM}}{c^2 r^{3/2}} + \frac{3a^2}{c^2 r^2} \\ \mathcal{D} &= \frac{1}{2\sqrt{r}} \int_{r_{\text{isco}}}^r \frac{x^2 c^2 - 6xGM + 8a\sqrt{xGM} - 3a^2}{\sqrt{x}(x^2 c^2 - 3xGM + 2a\sqrt{xGM})} dx \end{aligned} \quad (18)$$

where  $r_{\text{isco}}$  is the root of the following equation for the case of a co-rotating disk:

$$1 - 6\frac{MGc}{r} + 8\frac{a}{c} \sqrt{\frac{MGc}{r^3}} - 3\frac{a}{cr} = 0 \quad (19)$$

Here we explore a simple example of a thin accretion disk with a constant vertical temperature profile (thus, Equation (12) is applicable) in the galactic center. We note that the accretion system around the supermassive black hole at the center of our galaxy



**Figure 3.** Shows the absolute magnitude of magnetic field generation rate as a function of radius from the last stable circular orbit ( $r_{\text{isco}}$ ) a Schwarzschild black hole with different values of  $M$  in  $M_{\odot}$  units and  $\dot{M}$  in ( $M_{\odot}/\text{year}$ ) units. The solid line corresponds to a Schwarzschild black hole with  $M = 4 \times 10^6 M_{\odot}$  and  $\dot{M} = 10^{-10} M_{\odot}/\text{year}$ . The dashed line corresponds to the same mass but with 10 times higher accretion rate. The dotted line is for a black hole with 10 times smaller mass but the same the accretion rate. At a fixed  $r/r_{\text{isco}}$  the battery is proportional to  $M^{-2}$  and is weakly dependent on accretion rate ( $\propto \dot{M}^{1/8}$ ). The radial turning point ( $r_{\text{turn}}$ ) is independent of both mass and accretion rate. We assume  $z = H$  in this calculation.

is probably best modeled as an ADAF (e.g., Narayan & Yi 1994; Narayan et al. 1995; Abramowicz et al. 1988), which are expected to result in an even larger field strength. We discuss ADAFs in section 5.

Figure 2 shows the magnetic field growth rate for a  $4 \times 10^6 M_{\odot}$  black hole with different spin parameters as a function of  $r/r_{\text{isco}}$ . The battery is stronger for black holes with larger spin parameter at a fixed  $r/r_{\text{isco}}$ .

We find that the Biermann battery in the thin accretion disk model is weakly dependent on accretion rate ( $\propto \dot{M}^{1/8}$  at  $z = H$ ) and proportional to  $M^{-2}$  at a fixed  $r/r_{\text{isco}}$ . This is illustrated in Figure 3. As in the case of protoplanetary disk, the generated seeds have opposite sign in the upper and lower half of the disk. The change of azimuthal sign of  $dB/dt$  at  $r_{\text{turn}}$  arises because  $T_e$  has a maximum at this radius which is predicted by the thin disk model (see Shafee et al. (2008); Noble et al. (2009); Kulkarni et al. (2011) for a discussion of the zero-torque condition at the ISCO and the effect it has on the viscous heating profile in a thin accretion disk). However,  $B_{\phi}$  changes its sign across the mid plane and this result is a robust prediction of the theory.

## 5 ADVECTION-DOMINATED ACCRETION FLOWS DISKS

In many accretion disks around black holes the viscously dissipated accretion energy can go into heating the accretion flow rather than being radiated away. This is the most important characterization of advection-dominated accretion flows (ADAFs) disks (see for recent review: Yuan & Narayan 2014). Following Narayan & Yi (1994) self-similar solution, for  $\alpha \ll 1$  and an axis-symmetric flow, we

express the disk profile using the following scaling relations:

$$c_s^2 = \frac{2}{3} \frac{\gamma - 1}{\gamma - 5/9} v_k^2 \quad (20)$$

$$v_r = - \left( \frac{\gamma - 1}{\gamma - 5/9} \right) \alpha v_k \quad (21)$$

$$\rho = - \frac{\dot{M}}{4\pi R H v_r}, \quad (22)$$

where the scale height  $H$  is along the  $z$  coordinate,  $v_r$  is the radial component of the velocity of the gas,  $v_k = \sqrt{GM/r}$  is the Keplerian velocity, and  $\gamma$  is the specific heat of the gas and its value is between  $4/3$  to  $5/3$ , for a radiation-pressure dominated and a non-relativistic gas-pressure dominated accretion flow, respectively. We assume a gas pressure dominated disk, i.e.,  $P = c_s^2 \rho$ , and the density of the gas relates to the electron number density via:  $n_e = \chi_e \rho / \mu$ , where  $\chi_e$  is the ionization fraction and  $\mu$  is the mean molecular mass. We adopt  $\mu = 1.18 m_p$  and  $\chi_e = 1$  where  $m_p$  is the mass of the proton. Using Equation (1) we find that

$$\frac{\partial \mathbf{B}}{\partial t} \sim \frac{c}{en_e} \frac{\partial \rho}{\partial H} \frac{\partial c_s^2}{\partial r} \hat{\phi} \sim \frac{\mu 6cGM(\gamma - 1)}{er^3(9\gamma - 5)} \hat{\phi}, \quad (23)$$

where in our last transition we have plugged in the relation in Equations (20)–(22). Note that in our ADAF model the scale height  $H$  describes the vertical component, and due to the “puffed up” nature of the disk we set  $H \sim r$ . This has simple scaling of:

$$\frac{\partial B}{\partial t} \sim 3.6 \times 10^6 \left( \frac{r}{r_{\text{sc}}} \right)^{-3} \left( \frac{M}{M_{\odot}} \right)^{-2} \frac{\gamma - 1}{(9\gamma - 5)} G/\text{sec}, \quad (24)$$

where  $r_{\text{sc}} = 2GM/c^2$  is the Schwarzschild radius. For a characteristic accretion time  $t_{\text{acc}} \sim r/v_r$  we find:

$$B \sim 17.9 \left( \frac{M_{\odot}}{M} \right) \left( \frac{r_{\text{sc}}}{r} \right)^{3/2} \frac{\gamma - 1}{(9\gamma - 5)} G, \quad (25)$$

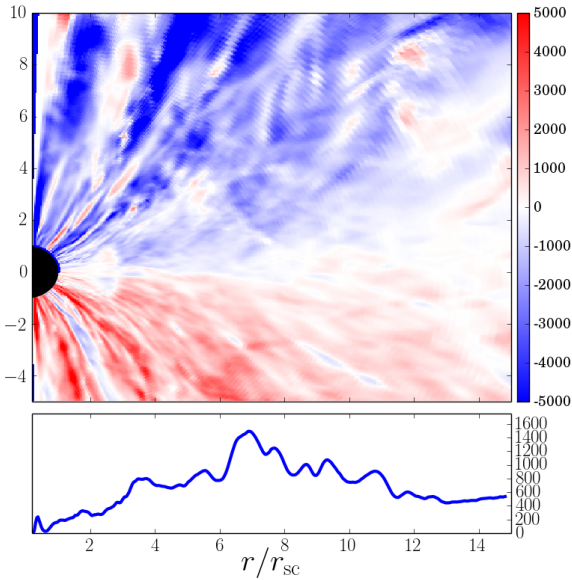
which is orders of magnitude smaller than the equipartition field in an ADAF which can be estimated as:

$$B_{\text{eq}} \sim 7.8 \times 10^7 G \frac{M_{\odot}}{M} \left( \frac{r_{\text{sc}}}{r} \right)^{5/4} \times \left( \frac{\dot{M}}{10^{-10} M_{\odot}/\text{yr}} \right)^{1/2} \left( \frac{1 - \gamma}{\alpha(5/9 - \gamma)} \right)^{1/2}. \quad (26)$$

From these relations we see that for extremely low accretion rates ( $\sim 2.3 \times 10^{-17} (r/r_{\text{sc}})^{1/4} M_{\odot} \text{ yr}^{-1}$ ) the two magnetic field strengths equate.

We now compare the estimates of the growth rate of the azimuthal component of the magnetic field due to the battery obtained above under simplifying assumptions, with rates calculated using a more detailed accretion flow structure produced by GRMHD simulations. For this purpose, we took the model of radiatively inefficient, two-temperature accretion flow presented and discussed in Sadowski et al. (2016) (model Rad8). This particular solution describes an accretion flow on a non-rotating,  $10 M_{\odot}$  BH with average accretion rate of the order of  $4 \times 10^{-9} \dot{M}_{\text{Edd}}$ . To calculate the battery-related growth of the magnetic field we took the time-averaged output of the simulation and applied Eq. (4). Results are presented in Fig 4.

The top panel presents the growth rate multiplied by radius cubed, i.e.,  $dB/dt(r/r_{\text{sc}})^3$ , on the poloidal plane of the simulation. The values are given in  $G/s$ . The most profound feature is the asymmetry against the equatorial plane. This reflects the fact that the gradients of density and temperature are symmetric with respect to the equatorial plane, and therefore their cross product changes



**Figure 4.** The growth rate of the azimuthal magnetic field,  $dB/dt(r/r_{sc})^3$ , given in  $G/s$ , in a GRMHD simulation of a radiatively inefficient accretion flow (model Rad8 from Sadowski et al. 2016) for a  $10 M_{\odot}$  BH. The top panel shows the growth rate in the poloidal plane.  $dB/dt$  changes signs over the equatorial plane because of symmetry in gradients of density and temperature. The bottom panel shows the average absolute value of  $dB/dt(r/r_{sc})^3$  averaged vertically over  $-1 < z/r < 1$ .

sign. For the same reason of disk structure symmetry, the growth rate at the equatorial plane is zero. Far from the plane, the field does not change significantly and reaches values of the order of  $dB/dt(r/r_{sc})^3 \approx 1000 G/s$ .

The bottom panel of Fig 4 shows the absolute value of the growth rate averaged vertically over the region  $-1 < z/r < 1$ . It confirms the typical value and shows that the growth rate scale well with the third power of radius. The average value should be compared to the analytical prediction based on Eq. (24) which gives, for  $10 M_{\odot}$ ,  $\gamma = 5/3$ , and  $\chi_e = 1$ ,

$$\frac{\partial B}{\partial t} \left( \frac{r}{r_{sc}} \right)^3 = 2400 G/s. \quad (27)$$

Having in mind that this estimate was derived under simplified assumptions about the vertical and radial structure of ADAFs, one should consider the agreement very good.

## 6 DISCUSSION

The generation of magnetic fields through the radiation pressure on the electrons in the inner edge of the accretion disks has been discussed in previous works (Contopoulos & Kazanas 1998; Bisnovatyi-Kogan et al. 2002; Contopoulos et al. 2015). In this letter we showed that large toroidal magnetic fields could naturally be generated through Biermann battery process starting initially from zero magnetic field. The vertical density profile of an accretion disk and the radial temperature profile which arises due to the balance between heating and cooling, lead to generation of toroidal seed magnetic field through the Biermann battery. These toroidal fields change their sign at the equatorial plane of the disk.

We considered few astrophysical examples.

- **Protoplanetary disks:** Assuming a simple disk profile, we found that a toroidal magnetic fields can naturally grow from initially not magnetized accretion disks in protoplanetary disks over the life time of the disk (at the order of few to tens of Gauss, see Figure 1). These fields are linearly proportional with the central mass and weakly dependent of the accretion rate. The generated seeds have opposite signs in the upper half ( $+\hat{\phi}$ ) and lower half ( $-\hat{\phi}$ ) of the disk. We also find that these fields change sign at a characteristic radius of  $r_{turn} = 49/36 R_*$ . Note that the radial change of sign at  $r_{turn}$  arises because  $T_e$  is maximum at this radius, as predicted by the thin disk model. However, this picture is disputed in the literature.

- **Thin accretion disk around a rotating BH:** We adopted a disk temperature and density profile, which includes relativistic corrections (following Novikov & Thorne 1973; Page & Thorne 1974; Doerr et al. 1996). The seeds have opposite sign in the upper and lower half of the disk and also change their sign in the azimuthal direction at  $r_{turn} = 2 \times r_{isco}$  for a Schwarzschild black hole. Larger BH spins yield smaller values of  $r_{turn}$  and larger rate of seed generation (see Figure 3). We note that the vanishing of  $T_e$  at  $r = r_{isco}$ , (as predicted by Shakura & Sunyaev 1973) is controversial because it comes from the zero-torque boundary condition at the ISCO and from ignoring advection. Thus, the behavior of sign flip may not be realistic. At a fixed  $r/r_{isco}$  the battery scales as  $M^{-2}$  and the battery is stronger for BHs with larger specific angular momentum. The relativistic disk profile leads to a weak dependency on the black hole accretion rate  $\propto \dot{M}^{1/8}$  (see Figure 3).

- **ADAF:** We also analyzed the generation of magnetic field in an ADAF, where our analytic estimates, based on Narayan & Yi (1994) self-similar solution, are in good agreement with GRMHD simulations (see Figure 4). Using Equation (25) we can estimate the magnetic field generated after one characteristic accretion time for the disk around the central BH in our galactic center. We find that the Biermann battery can naturally give rise to  $\sim 5 \times 10^{-6} G$  for the supermassive BH at the center of our galaxy within one accretion timescale. After one year of operation this mechanism can naturally reach a magnitude of 7 G very near the accretion radius ( $r \sim r_{sc}$ ), which is at the order of the few G estimations of the magnetic field strength in the vicinity of the galactic center (e.g., Mościbrodzka et al. 2009).

We note that while there is ample discussion in the literature about the role of poloidal field (e.g., Narayan et al. 2003) in subsequent evolution of accretion disks, there is little discussion about  $\hat{B}_{\phi}$ . Our study shows that this field naturally evolves in disks and therefore it would be interesting to simulate the role that Biermann-generated  $\hat{B}_{\phi}$  might play in the evolution of the accretion disks. This may also help to better understand the role of azimuthal sign reversal in the upper and lower half of the disk and potentially at  $r = r_{turn}$ .

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